A numerical solution is obtained for the two-dimensional nonsteady problem of the motion of detonation products from a cylindrical high-explosive charge enclosed in a shell with the initiation of detonation at a central point in the end of the charge. The detonation products propagate in vacuum. The strength of the shell is not considered. A three-term equation of state is used for the detonation products.

Consider a high-explosive charge in a cylindrical shell. A spherical detonation wave is initiated at a central point of the open right end. The detonation products propagate out into a vacuum. After arrival of the detonation wave at the open left end the detonation products begin to propagate to the left of the charge, and a rarefaction wave propagates from the end through the detonation products.

The motion of the detonation products is described by a system of gasdynamical equations in Eulerian variables, which is closed by the three-term equation of state proposed in [1]. We have previously solved the problem of the motion of detonation products in a she11 for the case of a plane detonation wave incident on an absolutely rigid wall [2]. A solution has been obtained in [3] for the two-dimensional gasdynamical problem associated with point ignition of a high-explosive charge in a cylindrical shell capped at the ends by thin cover plates.

We now investigate a charge of high-explosive pentolite (50:50 alloy of TNT and PETN) with an initial density $\rho_{0}=1.65 \mathrm{~g} / \mathrm{cm}^{3}$, heat of explosive reaction $Q=0.0536 \mathrm{Mbar} \cdot \mathrm{cm}^{3} / \mathrm{g}$, and detonation rate $D=0.7655 \mathrm{~cm} / \mu \mathrm{sec}$. The defining parameters of the problem for the specified equation of state of the detonation products are

$$
\mu=m / M, \lambda=L / R_{0}
$$

Here $m$ is the mass of the charge, $M$ is the mass of the shell, $L$ is the length, and $R_{0}$ is the initial radius of the charge. The motion of the shell obeys the law

$$
d M d \mathbf{W} / d t=p d S \mathbf{n}
$$

Here $W$ is the velocity vector of an element of mass $d M, p$ is the pressure of the detonation products on the inner surface $d S$ corresponding to the mass $d M$, and $n$ is the unit normal vector into the shell. The problem is solved in dimensionless variables, which are introduced to leave invariant the form of the gasdynamical equations and equation of state:

$$
\begin{aligned}
& \rho^{\prime}=\rho / \rho_{0}, u^{\prime}=u / D, v^{\prime}=v / D, p^{\prime}=p / \rho_{0} D^{2}, c^{\prime}=c / D \\
& e^{\prime}=e / D^{2}, r^{\prime}=r / R_{0}, z^{\prime}=z / R_{0}, t^{\prime}=t D / R_{0}
\end{aligned}
$$

Here $\rho$ is the density, $u$ and $v$ are the radial and axial components of the mass flow rate of the detonation products, $p$ is the pressure, $c$ is the speed of sound, $e$ is the internal energy, $r$ and $z$ are the radial and axial coordinates, and $t$ is the time.

The equation of motion of the shell, written for the velocity components in dimension-

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less variables, has the form (we now drop the primes for all time)
$$
d U / d t=2 p R \mu, d V / d t=2 p R \mu \operatorname{tg} \gamma
$$

Here $U$ and $V$ are the vertical and horizontal components of the shell velocity vector, and $\gamma$ is the angle between the vertical and the normal to the shell surface.

The boundary condition on the shell has the form

$$
W_{n}=w_{n}
$$

where $W_{n}$ and $W_{n}$ are the projections of the velocity vectors of the shell and detonation products onto the normal to the shell.

The boundary conditions at the advancing front of the escaping detonation products have the form

$$
p=0, \rho=0
$$

By the axial symmetry of the problem the radial velocity component is equal to zero on the axis. Prior to arrival of the detonation wave at the free end the boundary conditions at the wavefront have the form

$$
p=p_{c-j}, \rho=\rho_{c-j}, w=w_{c-j}
$$

Here $p_{c_{-j}}, \rho_{c-j}, W_{c-j}$ are the values of the parameters at the Chapman-Jouguet point. After arrival of the detonation wave at the left end the detonation products propagate to the left.

The initial conditions are specified on the basis of a numerical solution of the twodimensional self-similar problem for the motion of detonation products in point ignition [4].

A finite-difference approximation is obtained for the gasdynamical equations by means of an implicit two-step second-order scheme. The analytical method and finite-difference
scheme are described in detail in [5]. The calculations were run on a BÉSM digital computer on a $50 \times 30$ grid.

The positions of the shell and gas cloud at different times are given in Fig. 1 ( $\lambda=$ 4 , $\mu=2$ ): 1) $t=2.9$; 2) 4.38 ; 3) 6.12 ; 4) detonation wavefront. The rapid escape of the detonation products causes a rapid drop in the pressure of the detonation products in the end zone, so that the part of the shell close to the end acquires relatively smaller accelerations and remains far from the central part.

The curvature of the front of the spherical detonation wave diminishes as it moves through the charge, and upon arrival at the free end it can be considered to be plane with a high degree of accuracy. The cloud of detonation products at the right end propagates weakly to the left of the end, i.e., the detonation products "backwash" slightly onto the she11. Isobars for $t=6.12$ at the instant of fully developed motion of the shell are given in Fig. 2 for various pressures: 1) $p=0.0015$; 2) 0.004 ; 3) 0.0025 ; 4) 0.0015 ; 5) 0.0002 ; 6) 0.000001 . The pattern of the isobars reflects the interaction of the end-on and lateral rarefaction waves. Figure 3 shows the streamlines of the detonation products at the same instant.

The dashed curve in Fig. 3 indicates the sonic line ( $c=w$ ). The results evince the presence of a small subsonic zone in the central flow region; the main flow region is supersonic. The subsonic zone shrinks as the process evolves.

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